INVESTIGATION OF ACCURACY OF THE NEWEST FUNCTION APPROXIMATION FOR THE FORCE GENERATED BY PNEUMATIC ARTIFICIAL MUSCLE

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ABSTRACT

The newest and most promising type of pneumatic actuators is the pneumatic artificial muscle (PAM). Different designs have been developed, but the McKibben muscle is the most popular and is made commercially available by different companies (e.g., Fluidic Muscle manufactured by Festo Company). The most often mentioned characteristic of PAMs is the force as a function of pressure and contraction. In this paper, our newest function approximation for the force generated by Fluidic Muscles is shown that can be generally used for different muscles made by Festo Company.

1. INTRODUCTION

Pneumatic artificial muscle is a membrane that will expand radially and contract axially when inflated, while generating high pulling force along the longitudinal axis. PAMs have different names in literature: Pneumatic Muscle Actuator, Fluid Actuator, Fluid-Driven Tension Actuator, Axially Contractible Actuator, Tension Actuator, etc. ([1] and [2]).

The working principle of pneumatic muscles is well described in [1], [2], [3], [4] and [5].

There are a lot of advantages of PAMs like the high strength, good power-weight ratio, low price, little maintenance needed, great compliance, compactness, inherent safety and usage in rough environments ([4] and [6]). The main disadvantage of these muscles is that their dynamic behaviour is highly nonlinear ([4], [7], [8], [9], [10] and [11]).

Many researchers have investigated the relationship of the force, length and pressure to find a good theoretical approach for the equation of force produced by pneumatic artificial muscles. Some of them report several mathematical models, but significant differences have been noticed between the theoretical and experimental results ([4], [6], [12], [13] and [14]). [15] proves the accuracy of fitting using mathematical method of statistics (correlation index R = 0.998-0.999), only, but it is valid for SAM (Shadow Air Muscle) made by Shadow Robot Company.
The force depends on length (contraction) under constant pressure. This force decreases with increasing position of the muscle and the muscle inflates. Our goal was to develop a precise approximation algorithm with minimum numbers of parameters for the force of different Fluidic Muscles.

The layout of this paper is as follows. Section 2 (Materials and Methods) presents the static modelling of PAMs and several force equations. Section 3 (Results and Discussion) compares the measured and calculated data. Finally, Section 4 (Conclusion and Future Work) gives the investigations we plan.

Fluidic Muscle type DMSP-20-400N-RM-RM (with inner diameter of 20 mm and initial length of 400 mm) produced by Festo Company is selected for our newest study (Figure 1).

2. MATERIALS AND METHODS

The general behaviour of PAMs with regard to shape, contraction and tensile force when inflated depends on the geometry of the inner elastic part and of the braid at rest (Figure 2), and on the materials used [1]. Typical materials used for the membrane construction are latex and silicone rubber, while nylon is normally used in the fibres. Figure 3 shows the materials of Fluidic Muscles.
Fig. 3. Materials of Fluidic Muscles

With the help of [4] and [6], the input and output (virtual) work can be calculated:

$$dW_{in} = p \cdot dV$$  \hspace{1cm} (1)

$dW_{in}$ can be divided into a radial and an axial component:

$$dW_{in} = 2 \cdot r \cdot \pi \cdot p \cdot ([dr] - r^2 \cdot \pi \cdot p \cdot [dl])$$ \hspace{1cm} (2)

The output work:

$$dW_{out} = -F \cdot dl$$ \hspace{1cm} (3)

By equating the virtual work components:

$$dW_{in} = dW_{out}$$ \hspace{1cm} (4)

Using (1) and (3):

$$F = p \cdot \frac{dV}{dl}$$ \hspace{1cm} (5)

Using (2) and (3):

$$F = 2 \cdot r \cdot \pi \cdot p \cdot \frac{dr}{dl} - r^2 \cdot \pi \cdot p$$ \hspace{1cm} (6)

On the basis of Figure 2:

$$\cos \alpha_0 = \frac{l_0}{h} \text{ and } \cos \alpha = \frac{l}{h}$$ \hspace{1cm} (7)

$$\sin \alpha_0 = \frac{2 \cdot \pi \cdot r_0 \cdot n}{h} \text{ and } \sin \alpha = \frac{2 \cdot \pi \cdot r \cdot n}{h}$$ \hspace{1cm} (8)

$$\frac{l}{l_0} = \frac{\cos \alpha}{\cos \alpha_0} \text{ and } \frac{r}{r_0} = \frac{\sin \alpha}{\sin \alpha_0}$$ \hspace{1cm} (9)
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\[ r = r_0 \sqrt{1 - \cos^2 \alpha} \sin \alpha_0 = r_0 \sqrt{1 - \left( \frac{l_0}{l_0} \cos \alpha_0 \right)^2} \]  \hspace{1cm} (10)

\[ \frac{dr}{dl} = -\frac{r_0 \cdot l_0^2 \cdot \cos^2 \alpha_0}{l_0^2 \cdot \sin \alpha_0} \cdot \frac{1}{\sqrt{1 - \left( \frac{l_0}{l_0} \cos \alpha_0 \right)^2}} \]  \hspace{1cm} (11)

By using (10) and (11) with (6) the force equation is found:

\[ F(p, \kappa) = r_0^2 \cdot \pi \cdot p \cdot (a \cdot (1 - \kappa)^2 - b) \]  \hspace{1cm} (12)

Where \( a = \frac{3}{l_0^2 \cdot \alpha_0} \), \( b = \frac{1}{\sin^2 \alpha_0} \), \( \kappa = \frac{l_0 - 1}{l_0} \), \( 0 \leq \kappa \leq \kappa_{\text{max}} \), and \( V \) the muscle volume, \( F \) the pulling force, \( p \) the applied pressure, \( r_0, l_0, \alpha_0 \) the initial inner radius and length of the PAM and the initial angle between the thread and the muscle long axis, \( r, l, \alpha \) the inner radius and length of the PAM and angle between the thread and the muscle long axis when the muscle is contracted, \( h \) the constant thread length, \( n \) the number of turns of thread and \( \kappa \) the contraction.

Consequently:

\[ F_{\text{max}} = r_0^2 \cdot \pi \cdot p \cdot (a - b), \text{if } \kappa = 0 \]  \hspace{1cm} (13)

and

\[ \kappa_{\text{max}} = 1 - \sqrt{\frac{b}{a}}, \text{if } F = 0 \]  \hspace{1cm} (14)

Equation (12) is based on the admittance of a continuously cylindrical-shaped muscle. The fact is that the shape of the muscle is not cylindrical on the end, but rather is flattened, accordingly, the more the muscle contracts, the more its active part decreases, so the actual maximum contraction ratio is smaller than expected [4]. Tondu and Lopez in [4] consider improving (12) with a correction factor \( \varepsilon \), because it predicts for various pressures the same maximal contraction. This new equation is relatively good for higher pressure (\( p \geq 200 \text{ kPa} \)). Kerscher et al. in [13] suggest achieving similar approximation for smaller pressure another correction factor \( \mu \) is needed, so the modified equation is:

\[ F(p, \kappa) = \mu \cdot r_0^2 \cdot \pi \cdot p \cdot (a \cdot (1 - \varepsilon \cdot \kappa)^2 - b) \]  \hspace{1cm} (15)

Where \( \varepsilon = a_\varepsilon \cdot e^{-p} - b_\varepsilon \) and \( \mu = a_\mu \cdot e^{-\kappa \cdot 40} - b_\mu \).
Significant differences between the theoretical and experimental results using (12) and (15) have been proved in [16] and [17]. To eliminate the differences new approximation algorithms with six and five unknown parameters have been introduced for the force generated by Fluidic Muscles:

\[ F(p, \kappa) = (a \cdot p + b) \cdot e^{c \kappa} + d \cdot p \cdot \kappa + e \cdot p + f \]  
(16)

\[ F(p, \kappa) = (p + a) \cdot e^{b \kappa} + c \cdot p \cdot \kappa + d \cdot p + e \]  
(17)

(16) can be generally used with high accuracy for different Fluidic Muscle independently from length and diameter under different values of pressure and (17) can be used with high accuracy for Fluidic Muscle with inner diameter of 20 mm, only.

The unknown parameters of (16) (a, b, c, d, e and f) and (17) (a, b, c, d and e) can be found by Solver in MS Excel 2010.

3. RESULTS AND DISCUSSION

Our newest analyses were carried out in MS Excel. Tensile force of Fluidic Muscle under different constant pressures is a function of muscle length (contraction). The force always drops from its highest value at full muscle length to zero at full inflation and position (Figure 4).

![Isobaric force-contraction diagram](image)

**Fig. 4. Isobaric force-contraction diagram**

Firstly, the measured data and calculated data using (12) were compared. As it is shown in Figure 5, there is only one intersection point between the measured and calculated results and no fitting.
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Fig. 5. Comparison of measured data and calculated data using (12)

\[ R^2 = 0.6165 \rightarrow R = 0.7852 \] correlation index proves the inaccurate fitting for the measured data (Figure 6).

In the interest of fitting the simulation was repeated with (15) (Figure 7). The coefficients \((a_s, b_s, a_e, \text{ and } b_e)\) of (15) were found using Solver in MS Excel. Values of unknown parameters of (15) are listed in Table 1.
Table 1. Values of unknown parameters of (15)

<table>
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<tr>
<th>Parameters</th>
<th>Values</th>
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<td>$b_k$</td>
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<td>$a_\varepsilon$</td>
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<tr>
<td>$b_\varepsilon$</td>
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Fig. 7. Comparison of measured data and calculated data using (15)

Figure 7 shows the measured and calculated results still do not fit. Better fitting was attained, but at a pressure of 0 kPa we still have a rather substantial inconsistency. This inconsistency can be seen in Figure 8.

Fig. 8. Relationship between the measured force and calculated force using (15)
To improve fitting quality under different values of pressure including 0 kPa new approximation algorithms have been introduced with 6 and 5 parameters. The unknown parameters of (16) and (17) can be found using Solver in MS Excel, too. Values of these unknown parameters are shown in Table 2. and Table 3.

**Table 2. Values of unknown parameters of (16)**

<table>
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<tbody>
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<td>d</td>
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<tr>
<td>e</td>
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<td>f</td>
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**Table 3. Values of unknown parameters of (17)**

<table>
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<tr>
<td>c</td>
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<td>d</td>
<td>296.3161465</td>
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<tr>
<td>e</td>
<td>-254.042387</td>
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The accurate fitting of (16) and (17) can be seen in Figure 9 and Figure 10.
As we can see we have consistent fitting even at a pressure of 0 kPa.

Figure 11 and Figure 12 illustrate the relationship between the measured force and calculated force. The $R^2 = 0.9995 \rightarrow R = 0.9997$ correlation index and $R^2 = 0.9993 \rightarrow R = 0.9996$ correlation index prove the tight relationship between them.
4. CONCLUSION AND FUTURE WORK

In this work new functions for the force generated by Festo Fluidic Muscle were introduced and the accuracy of these approximation algorithms was proved. The investigations were carried out in MS Excel environment. Our main aim is to develop a new general mathematical model for pneumatic artificial muscles applying our new models and results.

REFERENCES


