ABSTRACT

The authors present the general aspects of theoretical modeling of gasoline injection, the principle modeling of some of the elements of gasoline injection, namely the gasoline pump model and the pressure regulator model. It is presented the general model of a spark engine with gasoline injection. The modeling of the motor cycles with spark engines and gasoline injection suggested by the authors is realized by executing a computer program for determining the variation of the three-dimensional and bi-dimensional parameters with the sub-programs: program for calculating the SIE (dependence according to \( n \) and \( \lambda \) at \( t_0=-35...+45^\circ C \) and \( p_o=1\cdot10^2 \text{ kPa} \)); program for calculating the SIE (dependence according to \( n \) and \( t_0 \) at \( \lambda=1 \) and \( p_o=1\cdot10^2 \text{ kPa} \)); program for calculating the SIE cycle with gasoline injection. It was realized a study for calculating the pressure from the admission gallery \( p_{ga} \) and the admission pressure \( p_a \).

Keywords: pressure regulator, valve, pulverization, duration of injection.

1. INTRODUCTION: GENERAL ASPECTS OF THE MODELLING

The theoretical and experimental research of the elements and injection systems of gasoline, represent the fundamental components of the analysis and synthesis for knowing the static and dynamic quality performances. The experimental analysis takes place mainly when the system is known, while the theoretic one when we design the system. The usage of computer in modeling the systems considerably enlarges the possibility for studying a of a great number of models in a short period of time, and then through simulation we can follow the answer when applying different entering signals [1].

In the analysis and the synthesis in dynamic running of the injection systems for gasoline we usually use a series of laws, theorems, fluid mechanic and mathematics.

About the mathematical means, we can say that in the majority of the cases the differential equations systems which describe the phenomena which take place inside the injection systems of the gasoline are non-linear. There is no universally applicable methodology for solving these non-linear equations, so we solve them through approximation, which allow the description of the phenomena to the prejudice of the quality of the simulation.

Among the most frequently used methods we can mention:
- Linearization of calculating relation around a functioning stationary point; linear analysis on parts; analysis in the plan of phases;
- Analysis with description function.

In order to illustrate the linearization method we will present Taylor’s formula for a function with two variables. With the help of this formula we can linearize any of the functions around a point [7]. Being given a function \( f(x,y) \), which in the neighborhood of \((a,b)\) has continuous partial derivates of \((n+1)\) order, according to Taylor’s formula, can be written:

\[
f(x,y) = f(ab) + \frac{1}{1!}[f_x(a,b) \cdot (x-a) + f_y(a,b)(y-b)] + \frac{1}{2!}[f_{xx}(a,b)(x-a)^2 + 2f_{xy}(a,b)(x-a)(y-b) + \nonumber \\
+ f_{yy}(a,b)(y-b)^2] + \ldots + \frac{1}{n!}(x-a)^n \cdot \frac{\partial^n f}{\partial x^n} + (y-b)^n \cdot \frac{\partial^n f}{\partial y^n} \]

\[= f(a,b) + R_n(x,y); \tag{1} \]

where: \( R_n(x,y) = \frac{1}{(n+1)!}[x-a]^n \cdot \frac{\partial^n f}{\partial x^n} + (y-b)^n \cdot \frac{\partial^n f}{\partial y^n} \)

Where: in the particular case \( a=b=0 \): we obtain Mac Laurin’s formula. Structurally, a system of gasoline injection from an engine represents a succession of energy conversions \( C_1, C_2, \ldots, C_5 \), Fig. 1. In the figure the blocks represent, in order: the electric motor drive EMD, the gasoline pump PB, control and adjustment
equipment ARC, (directional equipment for flow regulation and pressure storage, filtering, injection), usually named the organs of the spark ignition engine ESL.

The electric engine is accumulator driven. It transforms the electric energy into mechanic energy, so here takes place the first conversion C1, and then the gasoline pump transforms the mechanic energy into pressure potential energy, in this way the second conversion takes place C2. The electromagnetic injector pulverizes and doses the gasoline in front of the admission valve realizing in this way the conversion of the pressure potential energy and electromagnetic energy into mechanic energy for moving the needle/indicator of the C3 injector. In the combustion chamber of the engine takes place a double energy conversion. When the fuel mixture burns the chemical energy of the fuel is converted into thermal energy C4, which is transformed into effective work (mechanic energy) and the C5 conversion takes place.

Compared with other systems in a classical acceptation having an entering amount $x_i$ and an outgoing amount $x_o$, Fig. 1. The system of gasoline injection is presented in systemic acceptation, like a multivariable system in which the component elements (blocks 1…n) are quadric-poles or sexa-poles, and the connection lines (poles) represent the information support of the variables, Fig. 2.

The global driving system supposes the existence of an electric amount (U-voltage, I-intensity), mechanic ($n_1$- the pump’s number of rotations); n (v)- number of rotations of the ESL; $M_1$ – the moment of the pump; $M(F)$- the moment of the ESL, and hydraulics ($Q_p$- the pump’s flow rate; $Q_M$ the air flow rate at the ESL; pp- pressure of the pump; pM- the pressure at the MAS) ordered in such a way according to the way of transmission of the energy or information. The amounts U, $n_1$, $Q_p$, $Q_M$, and n(v) are called direct variables or movement variables, through which the frequency characteristic of the system is made, while $M(F)$; $p_M$; $p_p$; $M_1$; I – are called effort variables, which supposes the power supply of the system for creating the force or the necessary moment for surmounting of resistant moments or forces from the engine. X and F’ are exterior amounts for command. From the global system presented in Fig. 1 we can distinguish the system made by the 2,3,4, blocks, which are in fact the injection system of the gasoline, having the mechanic connections at entrance $n_1$ and $M_1$; and n(v) and M(F) with four energy conversions at the exit. This subsystem is analyzed on each of its components.

\[ W_{aer} = \frac{p_g - p_o}{\rho} + \left(1 + \varepsilon_{ga}\right) \frac{W_{ga}^2}{2}; \quad W_{aer} = 0. \]   \[ p_g = p_o - \frac{1}{2} \left(1 + \varepsilon_{ga}\right) \frac{p_g - p_o}{\rho} \left(1 + \varepsilon_{ga}\right) \frac{W_{ga}^2}{2}; \quad W_{aer} = 0. \]   \[ p_g = p_o - (1 + \varepsilon_{ga}) \frac{W_{ga}^2}{2} = p_o - (1 + \varepsilon_{ga}) \frac{W_{ga}^2}{2} \]
Where:
\( \rho_0 = \rho_{\text{air}} \) is the density of the air (considered constant); \( \rho = \gamma/g \); \( p_0 \) – the pressure from the entering section in the system; \( p_{ga} \) - pressure from the admission gallery; \( W_{ga} \) – the air flow speed through the admission gallery; \( \xi_{ga} \) - the coefficient of gas dynamic resistance of the admission gallery.

Applying the equation of continuity between the 1 – 1 and 2 – 2 sections it results:

\[
W_{ga} \cdot A_s = W_a \cdot A_c; \quad A_s = \pi d_o^2/4; \quad V_s = A_c \cdot S \Rightarrow A_c = \frac{V_s}{S};
\]

\( S \) - the stroke of the piston [m];

\[
W_a = W_{pm} = \frac{1}{30} \cdot S \cdot n; \quad W_{ga} = W_a \cdot \frac{A_c}{A_s} = \frac{1}{30} \cdot S \cdot n \cdot \frac{V_s}{S} \cdot \frac{1}{\pi d_o^2};
\]

\[
W_{ga} = \frac{4}{30 \cdot \pi} \cdot n \cdot \frac{V_s}{d_o^2}; \quad p_{ga} = p_0 - \left(1 + \xi_{ga}\right) \frac{\rho_{\text{aer}}}{2} \left(\frac{4}{30 \pi}\right)^2 \cdot n^2 \left(\frac{V_s}{d_o^2}\right)^2;
\]

A) The case when the density of the engine fluid \( \rho \) is constant

\[
p_{ga} = p_0 - k_1 \left(1 + \xi_{ga}\right) n^2 \left(\frac{V_s}{d_o^2}\right)^2;
\]

Where:
\[ k_1 = \frac{\rho_{\text{aer}}}{2} \left(\frac{4}{30 \pi}\right)^2 \]

In which: \( A_s \) – the area of the free section near the valve; \( V_s \) – unitary capacity; \( S \) – the stroke of piston; \( A_c \) – the area of transversal section of the cylinders; \( W_a \) - the air speed in the cylinder; \( W_{pm} \) – medium speed of the piston; \( d_o \) – diameter of the admission gallery; \( n \) – the number of rotations of the crank shaft;

We write the equation of Bernoulli between the sections 1 – 1 and 2 – 2 [4]:

\[
\frac{p_{ga}}{\gamma} + \frac{W_{ga}^2}{2g} = p_0 + \frac{1}{1 + \xi_{ga}} \frac{W_a^2}{2g};
\]

\[
p_a = p_{ga} + \frac{\gamma}{2g} W_{ga}^2 - \left(1 + \xi_{ga}\right) \frac{\gamma}{2g} W_a^2
\]

Where: \( \xi_{ga} = c_a \) – the coefficient of gas dynamic resistance of the admission way:

\[
W_{ga} = \frac{4}{30 \pi} \cdot n \cdot \frac{V_s}{d_o^2}; \quad W_a = \frac{S \cdot n}{30}; \quad S = \frac{V_s}{A_c} = \frac{V}{\pi D^2} = \frac{4V}{\pi D^2}; \quad W_a = \frac{4}{30 \pi} \cdot n \cdot \frac{V_s}{D^2};
\]
\[ p_a = p_\text{ga} + \frac{\rho_{\text{ao}}}{2} \cdot \left( \frac{4}{30\pi} \right)^2 \cdot n^2 \cdot \left( \frac{V_s}{D} \right)^2 \cdot (1 + \xi_a) \frac{\rho_{\text{ao}}}{2} \cdot \left( \frac{4}{30\pi} \right)^2 \cdot n^2 \cdot \left( \frac{V_s}{D} \right)^2; \]

\[ p_a = p_\text{ga} - k_1 \cdot n^2 \cdot V_s \left[ (1 + \xi_a) \frac{1}{D^4} - \frac{1}{d_o^4} \right]. \]

It results, the expression of the admission pressure:

\[ p_a = p_\text{ga} - k_1 \cdot n^2 \cdot V_s \left[ (1 + \xi_a) \frac{d_o^4}{D^4} - 1 \right]; \]

if: \[ (1 + \xi_a) \frac{d_o^4}{D^4} - 1 \geq 0; \quad (1 + \xi_a) \frac{d_o^4}{D^4} > 1; \quad \Rightarrow \xi_a > \frac{1}{\left( \frac{d_o}{D} \right)^4} - 1. \]

For the Dacia Logan vehicle the ratio \( \frac{d_o}{D} = 0.42; \xi_a \) - the coefficient of gas dynamic resistance of the admission way;

\[ \xi_a > \frac{1}{0.42^4} - 1; \xi_a > 32. \] [7]

By using gasoline injection, the gas dynamic resistance of the admission way is smaller than at the classical engines, where the carburetor and its diffuser introduce a gas dynamic resistance 8-10 times greater.

B) The case when the density of engine fluid \( \rho \) is variable [3].

Bernoulli’s relation between the section 0 – 0 and 2 – 2 in this case becomes:

\[ p_o = p_a + (1 + \xi_a) \frac{\rho_a}{2} \cdot W_o^2; \quad \rho_a = \frac{p_a}{RT_a}; \]

\[ W_o = W_p = \frac{n}{30} \cdot S = \frac{n}{30} \cdot \frac{V_s}{Ac} = \frac{4}{30\pi} \cdot \frac{V_s}{D^2} \cdot n; \]

\[ a_{sa} = \sqrt{k_a \cdot RT_a} \Rightarrow RT_a = \frac{a_{sa}^2}{k_a}; \quad \rho_a = \frac{k_a}{a_{sa}^2} \cdot p_a; \]

\[ p_o = p_a + (1 + \xi_a) \frac{k_a}{2} \cdot \frac{1}{a_{sa}^2} \cdot p_a \cdot \left( \frac{4}{30\pi} \right)^2 \cdot \left( \frac{V_s}{D^2} \right)^2 \cdot n^2; \]

\[ p_o = p_a \left[ 1 + (1 + \xi_a) \frac{k_a}{2} \left( \frac{4}{30\pi} \right)^2 \cdot \left( \frac{V_s}{D^2} \right)^2 \cdot \left( \frac{n}{a_{sa}} \right)^2 \right]; \]

\[ p_a = \frac{1}{1 + (1 + \xi_a) \frac{k_a}{2} \left( \frac{4}{30\pi} \right)^2 \cdot \left( \frac{V_s}{D^2} \right)^2 \cdot \left( \frac{n}{a_{sa}} \right)^2} \cdot p_o = k_1 \cdot p_o, \]

Where: \( k_1 = \frac{1}{1 + (1 + \xi_a) \frac{k_a}{2} \left( \frac{4}{30\pi} \right)^2 \cdot \left( \frac{V_s}{D^2} \right)^2 \cdot \left( \frac{n}{a_{sa}} \right)^2}; \]

Where:

- \( a_{sa} \) – the speed of the sound in the fluid at the admission in the cylinders; \( k_a \) – the medium adiabatic exponent of the admission;
- \( p_a \) – admission pressure.
Bernoulli’s equation between section 0 – 0 and 1 – 1 will be:

\[ p_o = p_{ga} + \left(1 + \xi_{ga}\right) \frac{\rho_{ga}}{2} \cdot W_{ga}^2; \quad \rho_{ga} = \frac{p_{ga}}{RT_{ga}} = \frac{p_{ga}}{RT_o}, \]  

(17)

Where:

\( p_{ga} \) – pressure from the admission gallery; \( T_o \) – temperature of the environment; \( T_{ga} \) – temperature in the admission gallery.

Applying the relation for the continuity of the mass flow we determine:

\[ W_{ga} \cdot A_s \cdot \rho_{ga} = W_a \cdot A_c \cdot \rho_a \]  

(18)

\( W_{ga} \) – the speed of gas flow in the admission gallery will be:

\[ W_{ga} = \frac{A_c}{A_s} \cdot \rho_a \cdot W_a = \frac{\pi D^2}{4} \cdot \frac{4}{\pi d_o^2} \cdot \frac{p_a}{RT_o} \cdot W_a; \]  

(19)

By introducing the \( W_{ga} \) in Bernoulli’s equation written for sections 0 – 0 and 1 – 1 it results: [5]

\[ p_o = p_{ga} + \left(1 + \xi_{ga}\right) \frac{p_{ga}}{2RT_o} \left( \frac{4}{30\pi} \right)^2 \cdot \frac{p_a}{p_{ga}} \cdot \left( \frac{T_o}{T_a} \right)^2 \cdot \left( \frac{V_s}{d_o^2} \right)^2 \cdot \rho_{oa} \cdot \left( \frac{n}{a_{oa}} \right)^2; \]  

(20)

\[ p_o = p_{ga} + \left(1 + \xi_{ga}\right) \frac{1}{2RT_a} \cdot \frac{p_a}{p_{ga}} \cdot \frac{T_o}{T_a} \cdot \left( \frac{V_s}{d_o^2} \right)^2 \cdot \rho_{oa} \cdot \left( \frac{n}{a_{oa}} \right)^2; \]  

We introduce the relation: \( RT_a = \frac{a_{ga}^2}{k_a} \),

\[ p_o \cdot p_{ga} = \rho_{oa} \cdot p_{ga}^2 + \left(1 + \xi_{ga}\right) \frac{k_a}{2a_{ga}^2} \left( \frac{4}{30\pi} \right)^2 \cdot \rho_{oa} \cdot \left( \frac{n}{a_{oa}} \right)^2 \cdot \left( k_1 \cdot p_o \right)^2 \cdot \left( \frac{V_s}{d_o^2} \right)^2; \]  

(21)

We write: \( p_a = k_1 \cdot p_o; \quad p_{ga}^2 - p_o \cdot p_{ga} + k_2 \cdot p_o^2 = 0. \)

We solve the second degree equation, with the unknown \( p_{ga} \), where:

\[ k_2 = \frac{k_a}{2} \cdot k_1^2 \left(1 + \xi_{ga}\right) \left( \frac{4}{30\pi} \right)^2 \cdot \frac{T_o}{T_a} \cdot \left( \frac{V_s}{d_o^2} \right)^2 \cdot \left( \frac{n}{a_{oa}} \right)^2; \quad k_2 > 0; \]  

(22)

\[ p_{ga} = \frac{p_o \pm \sqrt{p_o^2 - 4k_2 \cdot p_o^2}}{2} = \frac{p_o \pm p_o \sqrt{1 - 4k_2}}{2} = \frac{p_o (1 \pm \sqrt{1 - 4k_2})}{2}, \quad k_2 > 0; \]

\[ \sqrt{1 - 4k_2} \geq 0 \Rightarrow 0 \leq k_2 \leq \frac{1}{4}, \quad k_2 = 0; \quad n = 0; \Rightarrow p_{ga} = \frac{p_o (1 + 1)}{2} = p_o. \]

The solution is:

\[ p_{ga} = \frac{1 + \sqrt{1 - 4k_2}}{2} \cdot p_o = k_3 \cdot p_o \]  

(23)
Where: $k_3 = \frac{1 + \sqrt{1 - 4k_2^2}}{2}$.

**Figure 4.** Variation of the pressure from the admission gallery $p_{ga}$ with the number of rotations $n$ and the temperature of the environment $t_0$

**Figure 5.** Variation of admission pressure $p_{a}$ with the number of rotations $n$ and temperature of the environment $t_0$

Fig. 4 presents the pressure variation from the admission gallery $p_{ga}$ with the number of rotations and temperature of the environment, and Fig. 5 presents the admission pressure variation $p_{a}$ with the number of rotations and temperature of the environment from the calculus program.
3. CONCLUSIONS

a. It was devised a personal model for calculating the pressure for the admission gallery pga and for the pressure at the end of admission pa.
b. It was devised the model for calculating the MAS parameters with the gasoline injection with the subprograms: program for calculus of MAS parameters (dependence after n and $\lambda$ at $t_0=-35...+45^\circ$C and $p_0=1\cdot10^2$ kPa);
The program for calculating the parameters of the SIE (dependence on n and $t_0$ at $\lambda=1$ and $p_0=1\cdot10^2$ Pa);
c. With the help of the calculus program, we figured the three-dimensional of the adiabatic and polytrophic coefficients, temperatures and pressure in the characteristic points of the cycle, the filling coefficient, the dosage, the heat supply of the capacity of the engine, the pressure growth ratio in the isochoric burning, the volume growth ratio in post burning, the technical-economical parameters of the engine and the length of the injection.

REFERENCES